CONSOLIDACIÓN

Ficha *Integración por partes*

1. **a)** $I=\int 3x\cos(2x-1)\,dx$. Si se toma:

$$\left.\begin{array}{l} f(x)=3x\Rightarrow f'(x)=3 \\ g'(x)=\cos(2x-1)\Rightarrow g(x)=\int\cos(2x-1)\,dx=\frac{1}{2}\text{sen}(2x-1)\end{array}\right\}\Rightarrow$$

$$I=3x\frac{1}{2}\text{sen}(2x-1)-\int\frac{3}{2}\text{sen}(2x-1)\,dx=\frac{3x}{2}\text{sen}(2x-1)+\frac{3}{4}\cos(2x-1)+C$$

b) $I=\int x\,\text{sen}(1-5x)\,dx$. Si se toma:

$$\left.\begin{array}{l} f(x)=x\Rightarrow f'(x)=1 \\ g'(x)=\text{sen}(1-5x)\Rightarrow g(x)=\int\text{sen}(1-5x)\,dx=\frac{1}{5}\cos(1-5x)\end{array}\right\}\Rightarrow$$

$$I=\frac{x}{5}\cos(1-5x)-\frac{1}{5}\int\cos(1-5x)\,dx=\frac{x\cos(1-5x)}{5}+\frac{1}{25}\text{sen}(1-5x)+C$$

c) $I=\int\frac{x}{2}\cos(2x)\,dx$

$$\left.\begin{array}{l} f(x)=\frac{x}{2}\Rightarrow f'(x)=\frac{1}{2} \\ g'(x)=\cos(2x)\Rightarrow g(x)=\int\cos(2x)\,dx=\frac{1}{2}\text{sen}(2x)\end{array}\right\}\Rightarrow$$

$$I=\frac{x}{2}\cdot\frac{1}{2}\text{sen}(2x)-\frac{1}{4}\int\text{sen}(2x)\,dx=\frac{x\,\text{sen}(2x)}{4}+\frac{1}{8}\cos(2x)+C$$

d) $I=\int 2xe^{3x-5}dx$. Si se toma:

$$\left.\begin{array}{l} f(x)=2x\Rightarrow f'(x)=2 \\ g'(x)=e^{3x-5}\Rightarrow g(x)=\int e^{3x-5}dx=\frac{1}{3}e^{3x-5}\end{array}\right\}\Rightarrow I=2x\cdot\frac{1}{3}e^{3x-5}-\int\frac{2}{3}e^{3x-5}dx=\frac{2x}{3}e^{3x-5}-\frac{2}{9}e^{3x-5}+C$$

e) $I=\int 5xe^{\frac{5x-1}{2}}dx$

$$\left.\begin{array}{l} f(x)=5x\Rightarrow f'(x)=5 \\ g'(x)=e^{\frac{5x-1}{2}}\Rightarrow g(x)=\int e^{\frac{5x-1}{2}}dx=\frac{2}{5}e^{\frac{5x-1}{2}}\end{array}\right\}\Rightarrow I=2x\cdot 2e^{\frac{5x-1}{2}}-\int 2e^{\frac{5x-1}{2}}dx=2xe^{\frac{5x-1}{2}}-\frac{4}{5}e^{\frac{5x-1}{2}}+C$$

f) $I=\int -2xe^{2-x}dx$

$$\left.\begin{array}{l} f(x)=-2x\Rightarrow f'(x)=-2 \\ g'(x)=e^{2-x}\Rightarrow g(x)=\int e^{2-x}dx=-e^{2-x}\end{array}\right\}\Rightarrow I=-2xe^{2-x}-\int 2e^{2-x}dx=2xe^{2-x}+2e^{2-x}+C$$

g) $I=\int(x+1)\ln x\,dx$

$$\left.\begin{array}{l} f(x)=\ln x\Rightarrow f'(x)=\frac{1}{x} \\ g'(x)=x+1\Rightarrow g(x)=\int(x+1)dx=x^2+x\end{array}\right\}\Rightarrow I=(x^2+x)\ln x-\int(x^2+x)\cdot\frac{1}{x}dx=(x^2+x)\ln x-\frac{x^2}{2}-x+C$$

h) $I = \int x \operatorname{arctg} x dx$

$$\left.\begin{array}{l} f(x) = \operatorname{arctg} x \Rightarrow f'(x) = \dfrac{1}{1+x^2} \\ g'(x) = x \Rightarrow g(x) = \int x dx = \dfrac{x^2}{2} \end{array}\right\} \Rightarrow I = \frac{x^2}{2}\operatorname{arctg} x - \int \frac{x^2}{2} \cdot \frac{1}{x^2+1} dx = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \frac{x^2}{1+x^2} dx$$

La segunda integral se calcula haciendo previamente la división:

$$\int \frac{x^2}{1+x^2} dx = \int \left(1 - \frac{1}{1+x^2}\right) dx = x - \operatorname{arctg} x + C$$

$$I = \frac{x^2}{2}\operatorname{arctg} x - \frac{x}{2} + \frac{1}{2}\operatorname{arctg} x + C$$

i) $I = \int x \ln(x^2+1) dx$

$$\left.\begin{array}{l} f(x) = \ln(x^2+1) \Rightarrow f'(x) = \dfrac{2x}{x^2+1} \\ g'(x) = x \Rightarrow g(x) = \int x dx = \dfrac{x^2}{2} \end{array}\right\} \Rightarrow I = \frac{x^2}{2}\ln(x^2+1) - \int \frac{x^2}{2} \cdot \frac{2x}{x^2+1} dx = \frac{x^2}{2}\ln(x^2+1) - \frac{1}{2}\int \frac{2x^3}{x^2+1} dx + C$$

La segunda integral se calcula haciendo previamente la división:

$$\int \frac{2x^3}{1+x^2} dx = \int \left(2x - \frac{2x}{1+x^2}\right) dx = x^2 - \ln(x^2+1) + C$$

$$I = \frac{x^2}{2}\ln(x^2+1) - \frac{x^2}{2} + \frac{1}{2}\ln|x^2+1| + C$$

2. **a)** $I = \int x^2 \cos(2x-1) x dx$

$$\left.\begin{array}{l} f(x) = x^2 \Rightarrow f'(x) = 2x \\ g'(x) = \cos(2x-1) \Rightarrow g(x) = \int \cos(2x-1) dx = \dfrac{1}{2}\operatorname{sen}(2x-1) \end{array}\right\} \Rightarrow I = \frac{x^2}{2}\operatorname{sen}(2x-1) - \int x\operatorname{sen}(2x-1) dx$$

La segunda integral se vuelve a calcular por partes:

$$\left.\begin{array}{l} f(x) = x \Rightarrow f'(x) = 1 \\ g'(x) = \operatorname{sen}(2x-1) \Rightarrow g(x) = \int \operatorname{sen}(2x-1) dx = -\dfrac{1}{2}\cos(2x-1) \end{array}\right\}$$

$$\int x\operatorname{sen}(2x-1)\, dx = -\frac{x}{2}\cos(2x-1) + \frac{1}{2}\int \cos(2x-1) dx = -\frac{x}{2}\cos(2x-1) + \frac{1}{4}\operatorname{sen}(2x-1)$$

$$I = \frac{x^2}{2}\operatorname{sen}(2x-1) + \frac{x}{2}\cos(2x-1) - \frac{1}{4}\operatorname{sen}(2x-1) + C$$

b) $I = \int x^2 e^{3x-5} dx$

$$\left.\begin{array}{l} f(x) = x^2 \Rightarrow f'(x) = 2x \\ g'(x) = e^{3x-5} \Rightarrow g(x) = \int e^{3x-5} dx = \dfrac{1}{3}e^{3x-5} \end{array}\right\} \Rightarrow I = \frac{1}{3}x^2 e^{3x-5} - \frac{2}{3}\int x e^{3x-5} dx$$

Se aplica de nuevo la integración por partes:

$$\left.\begin{array}{l} f(x) = x \Rightarrow f'(x) = 1 \\ g'(x) = e^{3x-5} \Rightarrow g(x) = \int e^{3x-5} dx = \dfrac{1}{3}e^{3x-5} \end{array}\right\} \Rightarrow \int x e^{3x-5} dx = \frac{1}{3}x e^{3x-5} - \frac{1}{3}\int e^{3x-5} dx = \frac{1}{3}x e^{3x-5} - \frac{1}{9}e^{3x-5}$$

$$I = \frac{1}{3}x^2 e^{3x-5} - \frac{2}{3}\left(\frac{1}{3}x e^{3x-5} - \frac{1}{9}e^{3x-5}\right) = \frac{1}{3}x^2 e^{3x-5} - \frac{2}{9}x e^{3x-5} + \frac{2}{27}e^{3x-5} + C$$

c) $I=\int (2x^2-1)\cos x dx \Rightarrow \left.\begin{array}{l} f(x)=2x^2-1 \Rightarrow f'(x)=4x \\ g'(x)=\cos x \Rightarrow g(x)=\int \cos x\, dx = \operatorname{sen} x \end{array}\right\} \Rightarrow I=(2x^2-1)\operatorname{sen} x-\int 4x \operatorname{sen} x dx$

La segunda integral se vuelve a calcular por partes:

$\left.\begin{array}{l} f(x)=4x \Rightarrow f'(x)=4 \\ g'(x)=\operatorname{sen} x \Rightarrow g(x)=\int \operatorname{sen} x\, dx=-\cos x \end{array}\right\} \Rightarrow \int 4x \operatorname{sen} x\, dx = -4x\cos x+\int 4\cos x\, dx=-4x\cos x+4\operatorname{sen} x$

$I=(2x^2-1)\operatorname{sen} x+4x\cos x-4\operatorname{sen} x+C$

d) $I=\int x^2\operatorname{sen}(1-5x)dx \Rightarrow \left.\begin{array}{l} f(x)=x^2 \Rightarrow f'(x)=2x \\ g'(x)=\operatorname{sen}(1-5x) \Rightarrow g(x)=\int \operatorname{sen}(1-5x)dx=\frac{1}{5}\cos(1-5x) \end{array}\right\}$

$I=\frac{x^2}{5}\cos(1-5x)-\frac{2}{5}\int x\cos(1-5x)dx$

La última integral se vuelve a calcular por partes:

$\left.\begin{array}{l} f(x)=x \Rightarrow f'(x)=1 \\ g'(x)=\cos(1-5x) \Rightarrow g(x)=\int \cos(1-5x)dx=-\frac{1}{5}\operatorname{sen}(1-5x) \end{array}\right\}$

$\int x\cos(1-5x)dx=-\frac{x\operatorname{sen}(1-5x)}{5}+\frac{1}{5}\int \operatorname{sen}(1-5x)dx=-\frac{x\operatorname{sen}(1-5x)}{5}+\frac{1}{25}\cos(1-5x)$

$I=\frac{x^2}{5}\cos(1-5x)+\frac{2x\operatorname{sen}(1-5x)}{25}-\frac{2}{125}\cos(1-5x)+C$

e) $I=\int x^3\operatorname{sen}(2x)dx \Rightarrow \left.\begin{array}{l} f(x)=x^3 \Rightarrow f'(x)=3x^2 \\ g'(x)=\operatorname{sen}(2x) \Rightarrow g(x)=\int \operatorname{sen}(2x)dx=-\frac{1}{2}\cos(2x) \end{array}\right\} \Rightarrow$

$I=-\frac{x^3}{2}\cos(2x)+\frac{3}{2}\int x^2\cos(2x)dx$

La última integral se vuelve a calcular por partes:

$\left.\begin{array}{l} f(x)=x^2 \Rightarrow f'(x)=2x \\ g'(x)=\cos(2x) \Rightarrow g(x)=\int \cos(2x)dx=\frac{1}{2}\operatorname{sen}(2x) \end{array}\right\} \Rightarrow \int x^2\cos(2x)dx=\frac{x^2\operatorname{sen}(2x)}{2}-\int x\operatorname{sen}(2x)dx$

$I=-\frac{x^3}{2}\cos(2x)+\frac{3x^2\operatorname{sen}(2x)}{4}-\frac{3}{2}\int x\operatorname{sen}(2x)dx$. Y nuevamente, la última integral se hace por partes:

$\left.\begin{array}{l} f(x)=x \Rightarrow f'(x)=1 \\ g'(x)=\operatorname{sen}(2x) \Rightarrow g(x)=\int \operatorname{sen}(2x)dx=-\frac{1}{2}\cos(2x) \end{array}\right\} \Rightarrow \int x\operatorname{sen}(2x)dx=-\frac{x\cos(2x)}{2}\operatorname{sen} x+\frac{1}{2}\int \cos(2x)dx$

$I=-\frac{x^3}{2}\cos(2x)+\frac{3x^2\operatorname{sen}(2x)}{4}+\frac{3x\cos(2x)}{4}-\frac{3}{8}\operatorname{sen}(2x)+C$

f) $I=\int (x^2-1)e^{2-x}dx \Rightarrow \left.\begin{array}{l} f(x)=x^2-1 \Rightarrow f'(x)=2x \\ g'(x)=e^{2-x} \Rightarrow g(x)=\int e^{2-x}dx=-e^{2-x} \end{array}\right\} \Rightarrow I=-(x^2-1)e^{2-x}+2\int xe^{2-x}dx$

Se aplica de nuevo la integración por partes:

$\left.\begin{array}{l} f(x)=x \Rightarrow f'(x)=1 \\ g'(x)=e^{2-x} \Rightarrow g(x)=\int e^{2-x}dx=-e^{2-x} \end{array}\right\} \Rightarrow \int xe^{2-x}dx=-xe^{2-x}+\int e^{2-x}dx=-xe^{2-x}-e^{2-x}$

$I=-(x^2-1)e^{2-x}-2xe^{2-x}-2e^{2-x}+C$

3. **a)** $I = \int x^3 \cos(1-x^2)\,dx$.

Aplicando el método por partes:

$$\left.\begin{array}{l} f(x) = x^2 \Rightarrow f'(x) = 2x \\ g'(x) = x\cos(1-x^2) \Rightarrow g(x) = \int x\cos(1-x^2)\,dx = -\frac{1}{2}\text{sen}(1-x^2) \end{array}\right\}$$

$$I = \frac{-x^2\,\text{sen}(1-x^2)}{2} + \int x\,\text{sen}(1-x^2)dx = -\frac{x^2\,\text{sen}(1-x^2)}{2} + \frac{\cos(1-x^2)}{2} + C$$

b) $$I = \int x^3 e^{x^2}dx \Rightarrow \left.\begin{array}{l} f(x) = x^2 \Rightarrow f'(x) = 2x \\ g'(x) = xe^{x^2} \Rightarrow g(x) = \int xe^{x^2}dx = \frac{1}{2}e^{x^2} \end{array}\right\} \Rightarrow I = \frac{x^2e^{x^2}}{2} - \int xe^{x^2}dx = \frac{x^2e^{x^2}}{2} - \frac{e^{x^2}}{2} + C$$

c) $$I = \int \frac{x^3e^{3x^2-1}}{3}dx = \frac{1}{3}\int x^3e^{3x^2-1}dx \Rightarrow \left.\begin{array}{l} f(x) = x^2 \Rightarrow f'(x) = 2x \\ g'(x) = xe^{3x^2-1} \Rightarrow g(x) = \int xe^{3x^2-1}dx = \frac{1}{6}e^{3x^2-1} \end{array}\right\}$$

$$I = \frac{x^2}{6}e^{3x^2-1} - \frac{1}{3}\int xe^{3x^2-1}dx = \frac{x^2}{6}e^{3x^2-1} - \frac{1}{18}e^{3x^2-1} + C$$

d) $$I = \int x^5\text{sen}(-5x^3)\,dx \Rightarrow \left.\begin{array}{l} f(x) = x^3 \Rightarrow f'(x) = 3x^2 \\ g'(x) = x^2\text{sen}(-5x^3) \Rightarrow g(x) = \int x^2\text{sen}(-5x^3)\,dx = \frac{1}{15}\cos(-5x^3) \end{array}\right\} \Rightarrow$$

$$I = \frac{x^3}{15}\cos(-5x^3) - \frac{1}{5}\int x^2\cos(-5x^3)\,dx = \frac{x^3}{15}\cos(-5x^3) + \frac{1}{75}\text{sen}(-5x^3) + C$$

Ficha ***Integración por cambio de variable***

1. **a)** $I=\int(6x^3-2x)(3x^4-2x^2)^{100}\,dx$.

$t=3x^4-2x^2\Rightarrow dt=(12x^3-4x)dx\Rightarrow(6x^3-2x)dx=\frac{1}{2}dt$

$I=\frac{1}{2}\int t^{100}dt=\frac{1}{2}\cdot\frac{t^{101}}{101}+C=\frac{1}{202}(3x^4-2x^2)^{101}+C$

b) $I=\int\frac{x^2-2x}{(x^3-3x^2-1)^7}dx$.

$t=x^3-3x^2-1\Rightarrow dt=(3x^2-6x)dx\Rightarrow(x^2-2x)dx=\frac{1}{3}dt$

$I=\frac{1}{3}\int t^{-7}dt=\frac{1}{3}\frac{t^{-6}}{-6}+C=-\frac{1}{18(x^3-3x^2-1)^6}+C$

c) $I=\int\frac{2x-3}{\sqrt{x^2-3x+7}}dx$. Se hace el cambio: $t=x^2-3x+7\Rightarrow dt=(2x-3)dx$

$I=\int t^{-\frac{1}{2}}dt=-2t^{\frac{1}{2}}+C=-2\sqrt{x^2-3x+7}+C$

d) $I=\int\frac{\cos\left(\sqrt{2x-1}\right)}{\sqrt{2x-1}}dx$**.** Si se hace: $t=\sqrt{2x-1}\Rightarrow dt=\frac{1}{\sqrt{2x-1}}dx$

$I=\int\cos t\,dt=\operatorname{sen}t+C=\operatorname{sen}\sqrt{2x-1}+C$

e) $\int e^{-5x}\cos(e^{-5x})dx$. Si se hace el cambio: $t=e^{-5x}\Rightarrow dt=-5e^{-5x}dx\Rightarrow e^{-5x}dx=-\frac{dt}{5}$

$I=-\frac{1}{5}\int\cos t\,dt=-\frac{1}{5}\operatorname{sen}t+C=-\frac{1}{5}\operatorname{sen}(e^{-5x})+C$

f) $I=\int e^{3x}\cos(2e^{3x}-3)dx$. Si se hace el cambio: $t=2e^{3x}-3\Rightarrow dt=6e^{3x}dx\Rightarrow e^{3x}dx=\frac{1}{6}dt$

$I=\frac{1}{6}\int\cos t\,dt=\frac{1}{6}\operatorname{sen}t+C=\operatorname{sen}(2e^{3x}-3)+C$

g) $I=\int 2x^2e^{2-x^3}dx$. Si se hace el cambio $t=2-x^3\Rightarrow dt=-3x^2dx=dt\Rightarrow x^2dx=-\frac{1}{3}dt$

$I=-\frac{2}{3}\int e^t dt=-\frac{2}{3}e^t+C=-\frac{2}{3}e^{2-x^3}+C$

h) $I=\int\frac{\ln^3(2x)}{x}dx$. Si se hace el cambio $t=\ln(2x)\Rightarrow dt=\frac{1}{x}dx$

$I=\int t^3dt=\frac{t^4}{4}+C=\frac{\ln^4(2x)}{4}+C$

2. **a)** $I=\int\frac{\sqrt{x}}{1+\sqrt{x}}dx$. Haciendo el cambio indicado: $t=\sqrt{x}\Rightarrow t^2=x\Rightarrow 2tdt=dx$

$$I=\int\frac{t}{1+t}2tdt=2\int\frac{t^2}{1+t}dt=2\int\left(t-1+\frac{1}{1+t}\right)dt=t^2-2t+2\ln(1+t)=x-2\sqrt{x}+2\ln\left(1+\sqrt{x}\right)+C$$ **b)**

$I=\int\frac{\sqrt{x}}{1+x}dx$. Haciendo el cambio indicado: $t=\sqrt{x}\Rightarrow t^2=x\Rightarrow 2tdt=dx$

$$I=\int\frac{t}{1+t^2}2tdt=2\int\frac{t^2}{1+t^2}dt=2\int\left(1-\frac{1}{1+t^2}\right)dt=2t-2\text{arctg}t+C=2\sqrt{x}-2\text{arctg}\sqrt{x}+C$$

3. $I=\int\frac{e^x}{\sqrt{1-e^{2x}}}dx$. Haciendo el cambio $t=e^x\Rightarrow dt=e^xdx$

$$I=\int\frac{dt}{\sqrt{1-t^2}}=\text{arcsen}\,t+C=\text{arcsen}\,e^x+C$$

4. $I=\int\frac{x}{\sqrt[3]{x-1}}dx$. Haciendo el cambio $t=\sqrt[3]{x-1}\Rightarrow t^3=x-1\Rightarrow 3t^2dt=dx$

$$I=\int\frac{(t^3+1)3t^2}{t}dt=\int(3t^4+3t)dt=\frac{3t^5}{5}+\frac{3t^2}{2}+C=\frac{3\sqrt[3]{(x-1)^5}}{5}+\frac{3\sqrt[3]{(x-1)^2}}{2}+C$$

5. $I=\int\frac{\text{sen}(3x)}{1+\cos^2(3x)}dx$. Haciendo el cambio: $t=\cos(3x)\Rightarrow dt=-3\text{sen}(3x)dx\Rightarrow \text{sen}(3x)dx=-\frac{1}{3}dt$

$$I=-\frac{1}{3}\int\frac{1}{1+t^2}dt=\text{arctg}t+C=\text{arctg}(\cos(3x))+C$$

Ficha *Aplicación práctica de la regla de Barrow*

1. **a)** $\int_0^3 (x^3-3x^2+x-1)\,dx = \left.\frac{x^4}{4}-x^3+\frac{x^2}{2}-x\right]_0^3 = \left(\frac{81}{4}-27+\frac{9}{2}-3\right)-0 = -\frac{21}{4}$

b) $\int_{-1}^1 \left(3x^4-2\sqrt[3]{x^2}+\frac{5}{x^2}\right)dx = \left.3\frac{x^5}{5}+\frac{6\sqrt[3]{x^5}}{5}-\frac{5}{x}\right]_{-1}^1 = \left(\frac{3}{5}+\frac{6}{5}-5\right)-\left(-\frac{3}{5}-\frac{6}{5}+5\right) = -\frac{32}{5}$

c) $\int_2^5 \left(x^2-3\sqrt{x-1}\right)dx = \left.\left(\frac{x^3}{3}-2\sqrt{(x-1)^3}\right)\right]_2^5 = \left(\frac{125}{3}-16\right)-\left(\frac{8}{3}-2\right) = 25$

d) $\int_{-1}^2 (x^2-3e^x)\,dx = \left.\left(\frac{x^3}{3}-3e^x\right)\right]_{-1}^2 = \left(\frac{8}{3}-3e^2\right)-\left(-\frac{1}{3}-\frac{3}{e}\right) = \frac{-3e^3+3e+3}{e}$

e) $\int_3^5 \frac{2x-3}{\sqrt{x^2-3x+1}}dx = \left.2\sqrt{x^2-3x+1}\right]_3^5 = 2\sqrt{11}-2$

f) $\int_1^2 \frac{x^2+3x+1}{x}dx = \left.\left(\frac{x^2}{2}+3x+\ln x\right)\right]_1^2 = (2+6+\ln 2)-\left(\frac{1}{2}+3+\ln 1\right) = \frac{9}{2}+\ln 2$

g) $\int_1^9 \frac{\sqrt{x}}{1+\sqrt{x}}dx = \left.\left(x-2\sqrt{x}+2\ln(1+\sqrt{x})\right)\right]_1^9 = (9-6+2\ln 4)-(1-2+2\ln 2)4+\ln 4$

h) $\int_{-1}^2 3xe^{4x-2}dx = \left.\frac{3xe^{4x-2}}{4}-\frac{3e^{4x-2}}{16}\right]_{-1}^2 = \left(\frac{6e^6}{4}-\frac{3e^6}{16}\right)-\left(-\frac{3e^{-6}}{4}-\frac{3e^{-6}}{16}\right) = \frac{21e^{12}+15}{16e^6}$

i) $\int_4^8 \frac{\ln(x-3)}{x-3}dx = \left.\frac{\ln^2(x-3)}{2}\right]_4^8 = \frac{\ln^2 5}{2}-\frac{\ln^2 1}{2} = \frac{\ln^2 5}{2}$

2. Puntos de corte: $x = 0$, .$x = 4$ Posición curva: $f(x)\geq 0$

$$A = -\int_{-2}^0 (x^2+4x)\,dx + \int_0^2 (x^2+4x)\,dx = \frac{16}{3}+\frac{32}{3} = 16 \text{ u}^2$$

3. Puntos de corte: $x = 0$, .$x = 1$

Posición curva: En $[0,1]$, $f(x)\leq 0$, mientras que en $[1,2]$, $f(x)\geq 0$

$$A = -\int_0^1 (x-\sqrt{x})\,dx + \int_1^2 (x-\sqrt{x})\,dx = -\left.\left(\frac{x^2}{2}-\frac{2\sqrt{x^3}}{3}\right)\right]_0^1 + \left.\left(\frac{x^2}{2}-\frac{2\sqrt{x^3}}{3}\right)\right]_1^2 =$$

$$= -\left(\frac{1}{2}-\frac{2}{3}\right)+\left(2-\frac{4\sqrt{2}}{3}\right)-\left(\frac{1}{2}-\frac{2}{3}\right) = \frac{7-4\sqrt{2}}{3} \text{ u}^2$$

4. Puntos de corte: $x = -1$, $x = 0$, .$x = 1$

Posición curva: En $[-2,-1]\cup[0,1]$ $f(x)\geq 0$, mientras que en $[-1,0]$, $f(x)\leq 0$

$$A = \int_{-2}^{-1} (-x^3+x)\,dx - \int_{-1}^0 (-x^3+x)\,dx + \int_0^1 (-x^3+x)\,dx = \left.\left(-\frac{x^4}{4}+\frac{x^2}{2}\right)\right]_{-2}^{-1} - \left.\left(-\frac{x^4}{4}+\frac{x^2}{2}\right)\right]_{-1}^0 + \left.\left(-\frac{x^4}{4}+\frac{x^2}{2}\right)\right]_0^1 =$$

$$= \left(-\frac{1}{4}+\frac{1}{2}\right)-(-4+2)-\left(0-\left(-\frac{1}{4}+\frac{1}{2}\right)\right)+\left(-\frac{1}{4}+\frac{1}{2}\right)-0 = \frac{11}{4} \text{ u}^2$$

Ficha: ***Áreas de recintos planos***

1. Puntos de corte: $x = 1$, $x = 4$. Posición curvas: g por encima de f

Área: $\int_1^4 (-2x^2 + 10x - 8)\,dx = \left(-2\frac{x^3}{3} + 5x^2 - 8x\right)\Big]_1^4 = \left(-\frac{128}{3} + 80 - 32\right) - \left(-\frac{2}{3} + 5 - 8\right) = 9$ u^2

2. Puntos de corte: $x = -2$, $x = 0$, .$x = 2$

Posición curvas: En $[-2,0]$, $f(x) \geq g(x)$, mientras que en $[0,2]$, $g(x) \geq f(x)$

Área: $\int_{-2}^0 (x^3 - 4x)\,dx + \int_0^2 (-x^3 + 4x)\,dx = \left(\frac{x^4}{4} - 2x^2\right)\Big]_{-2}^0 + \left(-\frac{x^4}{4} + 2x^2\right)\Big]_0^2 = (0 - (-4)) + (-4 + 8) = 8$ u^2

3. Puntos de corte: $x = 0$, $x = 4$. Posición curvas: g por encima de f

Área: $\int_0^4 (2\sqrt{x} - x)\,dx = \left(\frac{4\sqrt{x^3}}{3} - \frac{x^2}{2}\right)\Big]_0^4 = \frac{32}{3} - 8 = \frac{8}{3}$ u^2

4. Puntos de corte: $x = -1$, $x = 1$, $x = 4$

Posición curvas: En $[-1,1]$, $f(x) \geq g(x)$, mientras que en $[1,4]$, $g(x) \geq f(x)$

Área: $\int_{-1}^1 (x^3 - 4x^2 - x + 4)\,dx + \int_1^4 (-x^3 + 4x^2 + x - 4)\,dx = \left(\frac{x^4}{4} - 4\frac{x^3}{3} - \frac{x^2}{2} + 4x\right)\Big]_{-1}^1 + \left(-\frac{x^4}{4} + 4\frac{x^3}{3} + \frac{x^2}{2} - 4x\right)\Big]_1^4 =$

$= \left(\frac{1}{4} - \frac{4}{3} - \frac{1}{2} + 4\right) - \left(\frac{1}{4} + \frac{4}{3} - \frac{1}{2} - 4\right) + \left(-64 + \frac{256}{3} + 8 - 16\right) - \left(-\frac{1}{4} + \frac{4}{3} + \frac{1}{2} - 4\right) = \frac{253}{12}$ u^2

5. Puntos de corte: $x = -2$, $x = 2$. Posición curvas: f por encima de g

Área: $\int_{-2}^2 \left(\frac{15}{x^2 + 1} - x^2 + 1\right)dx = 2\int_0^2 \left(\frac{15}{x^2 + 1} - x^2 + 1\right)dx = 2\left(15\text{arctg}x - \frac{x^3}{3} + x\right)\Big]_0^2 = 2\left(15\text{arctg}2 - \frac{8}{3} + 2\right)$ u^2

6. Puntos de corte: $x = 0$, $x = 2$.

Área: $\int_0^2 \frac{x}{e^x}dx = \left(-\frac{(x+1)}{e^x}\right)\Big]_0^2 = -\frac{3}{e^2} + 1 = \frac{e^2 - 3}{e^2}$ u^2

PROFUNDIZACIÓN

Ficha: *Integrales cíclicas*

1. **a)** $I=\int \operatorname{sen}(2x)\,e^{3x}\,dx$. Si se toma:

$$\left.\begin{array}{l} f(x)=\operatorname{sen}(2x)\Rightarrow f'(x)=2\cos(2x) \\ g'(x)=e^{3x}\Rightarrow g(x)=\int e^{3x}dx=\frac{1}{3}e^{3x}\end{array}\right\}\Rightarrow$$

$$I=\frac{1}{3}e^{3x}\operatorname{sen}(2x)-\int\frac{1}{3}e^{3x}2\cos(2x)\,dx=\frac{1}{3}e^{3x}\operatorname{sen}(2x)-\frac{2}{3}\int e^{3x}\cos(2x)\,dx$$

b) Si se toma:

$$\left.\begin{array}{l} f(x)=\cos(2x)\Rightarrow f'(x)=-2\operatorname{sen}(2x) \\ g'(x)=e^{3x}\Rightarrow g(x)=\int e^{3x}dx=\frac{1}{3}e^{3x}\end{array}\right\}\Rightarrow$$

$$\int e^{3x}\cos(2x)e^{3x}\,dx=\frac{1}{3}e^{3x}\cos(2x)+\frac{2}{3}\int e^{3x}\operatorname{sen}(2x)dx$$

c) Aparece, de nuevo, la integral inicial que se solicita resolver.

d) $$\int e^{3x}\operatorname{sen}(2x)dx=\frac{1}{3}e^{3x}\operatorname{sen}(2x)-\frac{2}{3}\int e^{3x}\cos 2x\,dx=\frac{1}{3}e^{3x}\operatorname{sen}(2x)-\frac{2}{3}\left[\frac{1}{3}e^{3x}\cos(2x)+\frac{2}{3}\int e^{3x}\operatorname{sen}(2x)\,dx\right]=$$

$$=\frac{1}{3}e^{3x}\operatorname{sen}(2x)-\frac{2}{9}e^{3x}\cos(2x)-\frac{4}{9}\int e^{3x}\operatorname{sen}(2x)\,dx$$

Es decir: $I=\frac{1}{3}e^{3x}\operatorname{sen}(2x)-\frac{2}{9}e^{3x}\cos(2x)-\frac{4}{9}I$

e) $$I=\frac{1}{3}e^{3x}\operatorname{sen}(2x)-\frac{2}{9}e^{3x}\cos(2x)-\frac{4}{9}I\Rightarrow I+\frac{4}{9}I=\frac{1}{3}e^{3x}\operatorname{sen}(2x)-\frac{2}{9}e^{3x}\cos(2x)\Rightarrow$$

$$\Rightarrow\frac{13}{9}I=\frac{1}{3}e^{3x}\operatorname{sen}(2x)-\frac{2}{9}e^{3x}\cos(2x)\Rightarrow I=\frac{9}{13}\left[\frac{1}{3}e^{3x}\operatorname{sen}(2x)-\frac{2}{9}e^{3x}\cos(2x)\right]\Rightarrow$$

$$\Rightarrow I=\frac{3}{13}e^{3x}\operatorname{sen}(2x)-\frac{2}{13}e^{3x}\cos(2x)+C$$

f) $$I'=\frac{9}{13}e^{3x}\operatorname{sen}(2x)+\frac{6}{13}e^{3x}\cos(2x)-\frac{6}{13}e^{3x}\cos(2x)+\frac{4}{13}e^{3x}\operatorname{sen}(2x)=\left(\frac{9}{13}+\frac{4}{13}\right)e^{3x}\operatorname{sen}(2x)=e^{3x}\operatorname{sen}(2x)$$

2. **a)** $I=\int\operatorname{sen}(\ln x)dx$

$$t=\ln x\Rightarrow x=e^t\Rightarrow dt=\frac{1}{x}dx\Rightarrow dx=e^tdt\Rightarrow I=\int e^t\operatorname{sen}t\,dt$$

b) $\int e^t\operatorname{sen}t\,dt=\frac{1}{2}[\operatorname{sen}t-\cos t]e^t+C$

c) $$\int\operatorname{sen}(\ln x)\,dx=\int\operatorname{sen}t\;e^t\,dt=\frac{1}{2}[\operatorname{sen}t-\cos t]e^t+C=\frac{1}{2}[\operatorname{sen}(\ln x)-\cos(\ln x)]e^{\ln x}+C=$$

$$=\frac{x}{2}[\operatorname{sen}(\ln x)-\cos(\ln x)]+C$$

d) $$\left(\frac{x}{2}[\operatorname{sen}(\ln x)-\cos(\ln x)]\right)'=\frac{1}{2}\operatorname{sen}(\ln x)-\frac{1}{2}\cos(\ln x)+\frac{x}{2}\cdot\frac{1}{x}(\cos(\ln x)+\operatorname{sen}(\ln x))=\operatorname{sen}(\ln x)$$

3. **a)** $t = \text{arcsen}\, x \Rightarrow x = \text{sen}\, t \Rightarrow dx = \cos t dt\} \Rightarrow \int \frac{x e^{\text{arcsen}\, x}}{\sqrt{1-x^2}} dx = \int \frac{sent \cdot e^{\text{arcsen}(\text{sen}\, t)}}{\sqrt{1-\text{sen}^2 t}} \cdot \cos t \; dt = \int \text{sen}\, t \cdot e^t dt$

b) $I = \int \text{sen}\, t \; e^t dt = \frac{1}{2}[\text{sen}\, t - \cos t] e^t + C = \frac{1}{2}[\text{sen}(\text{arcsen}\, x) - \cos(\text{arcsen}\, x)] e^{\text{arcsen} x} + C =$

$= \frac{1}{2}(x - \cos(\text{arcsen}\, x)) e^{\text{arcsen}\, x} + C$

Ficha: ***Integrales de potencias de las funciones trigonométricas***

1. a) $\cos 2\alpha = \cos^2\alpha - \text{sen}^2\alpha = 1 - \text{sen}^2\alpha - \text{sen}^2\alpha = 1 - 2\text{sen}^2\alpha \Rightarrow 2\text{sen}^2\alpha = 1 - \cos 2\alpha \Rightarrow \text{sen}^2\alpha = \dfrac{1-\cos 2\alpha}{2}$

$\cos 2\alpha = \cos^2\alpha - \text{sen}^2\alpha = \cos^2\alpha - 1 + \cos^2\alpha = -1 + 2\cos^2\alpha \Rightarrow 2\cos^2\alpha = 1 + \cos 2\alpha \Rightarrow \cos^2\alpha = \dfrac{1+\cos 2\alpha}{2}$

b) $\int \text{sen}^2 x dx = \int \dfrac{1-\cos 2x}{2} dx = \dfrac{1}{2}\int (1-\cos 2x) dx = \dfrac{1}{2}\left[x - \dfrac{1}{2}\int 2\cos 2x dx\right] = \dfrac{1}{2}x - \dfrac{1}{4}\text{sen}\, 2x + C$

$\int \cos^2 x dx = \int \dfrac{1+\cos 2x}{2} dx = \dfrac{1}{2}\int (1+\cos 2x) dx = \dfrac{1}{2}\left[x + \dfrac{1}{2}\int 2\cos 2x dx\right] = \dfrac{1}{2}x + \dfrac{1}{4}\text{sen}\, 2x + C$

c) $\int \text{sen}^2 3x dx = \int \dfrac{1-\cos 6x}{2} dx = \dfrac{1}{2}\int (1-\cos 6x) dx = \dfrac{1}{2}\left[x - \dfrac{1}{6}\int 6\cos 6x dx\right] = \dfrac{1}{2}x - \dfrac{1}{12}\text{sen}\, 6x + C$

$\int \cos^2 2x dx = \int \dfrac{1+\cos 4x}{2} dx = \dfrac{1}{2}\int (1+\cos 4x) dx = \dfrac{1}{2}\left[x + \dfrac{1}{4}\int 4\cos 4x dx\right] = \dfrac{1}{2}x + \dfrac{1}{8}\text{sen}\, 4x + C$

2. a) $\cos x = t \Rightarrow -\text{sen}\, x dx = dt$

$\int \text{sen}^3 x dx = \int \text{sen}\, x \cdot \text{sen}^2 x dx = \int \text{sen}\, x (1-\cos^2 x) dx = -\int (1-\cos^2 x)(-\text{sen} x) dx = -\int (1-t^2) dt =$

$= -t + \dfrac{1}{3}t^3 + C = -\cos x + \dfrac{1}{3}\cos^3 x + C$

b) $\text{sen}\, x = t \Rightarrow \cos x dx = dt$

$\int \cos^3 x dx = \int \cos x \cdot \cos^2 x dx = \int \cos x (1-\text{sen}^2 x) dx = \int (1-\text{sen}^2 x)\cos x dx = \int (1-t^2) dt =$

$= t - \dfrac{1}{3}t^3 + C = \text{sen}\, x - \dfrac{1}{3}\text{sen}^3 x + C$

3. a) $\text{sen}^4 x = (\text{sen}^2 x)^2 = \left(\dfrac{1-\cos 2x}{2}\right)^2 = \dfrac{1+\cos^2 2x - 2\cos 2x}{4} = \dfrac{1 + \dfrac{1+\cos 4x}{2} - 2\cos 2x}{4} = \dfrac{3+\cos 4x - 4\cos 2x}{8}$

$\int \text{sen}^4 x dx = \int \left(\dfrac{3+\cos 4x - 4\cos 2x}{8}\right) dx = \dfrac{3x}{8} + \dfrac{\text{sen}\, 4x}{32} - \dfrac{\text{sen}\, 2x}{4} + C$

b) $\cos^4 x = (\cos^2 x)^2 = \left(\dfrac{1+\cos 2x}{2}\right)^2 = \dfrac{1+\cos^2 2x + 2\cos 2x}{4} = \dfrac{1 + \dfrac{1+\cos 4x}{2} + 2\cos 2x}{4} = \dfrac{3+\cos 4x + 4\cos 2x}{8}$

$\int \cos^4 x dx = \int \left(\dfrac{3+\cos 4x + 4\cos 2x}{8}\right) dx = \dfrac{3x}{8} + \dfrac{\text{sen}\, 4x}{32} + \dfrac{\text{sen}\, 2x}{4} + C$

c) $\int \text{sen}^4 5x dx = \int (\text{sen}^2 5x)^2 dx = \int \left(\dfrac{1-\cos 10x}{2}\right)^2 dx = \int \dfrac{1+\cos^2 10x - 2\cos 10x}{4} dx =$

$= \int \dfrac{1 + \dfrac{1+\cos 20x}{2} - 2\cos 10x}{4} dx = \int \dfrac{3 + 2\cos 20x - 4\cos 10x}{8} dx = \dfrac{3x}{8} + \dfrac{\text{sen}\, 20x}{80} - \dfrac{\text{sen}\, 10x}{20} + C$