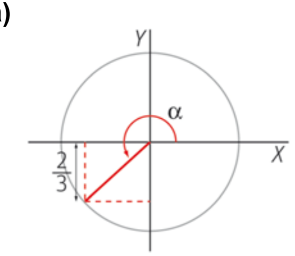
CONSOLIDACIÓN

Ficha: Dibujando ángulos

1. a)



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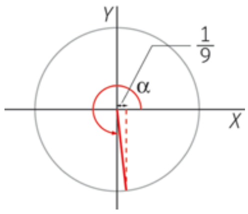

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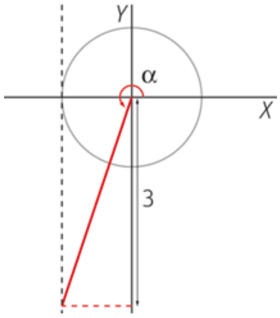

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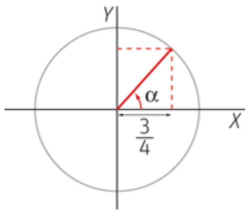

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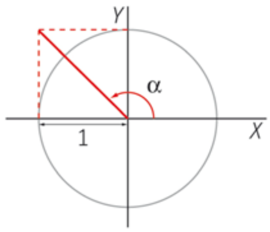

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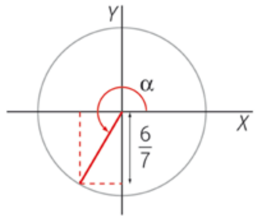

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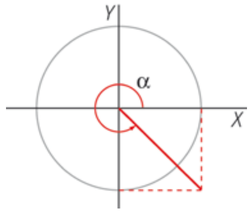

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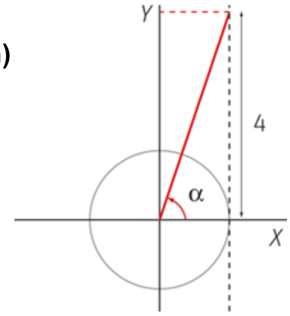
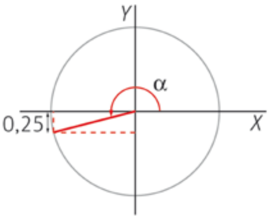


2. a)



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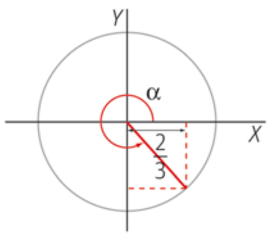

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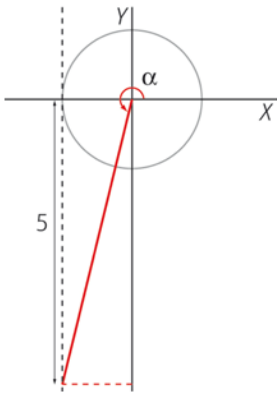

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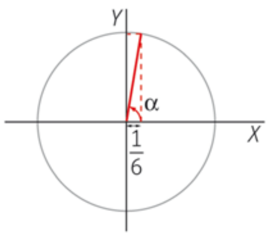

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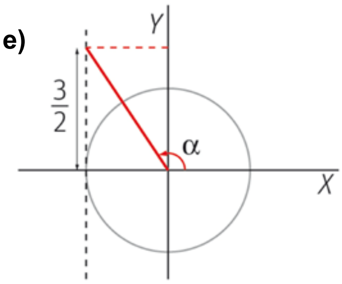

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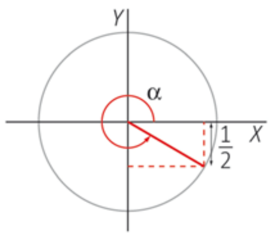

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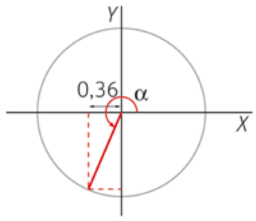

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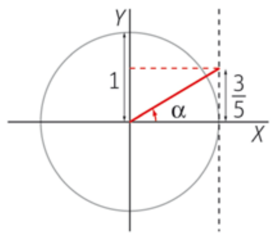

Ficha: Reducción al primer cuadrante

1.

	secante	cosecante	cotangente
$180^o - \alpha$	$-\sec \alpha$	$\text{cosec}\ \alpha$	$-\text{cotg}\ \alpha$
$-\alpha$	$\sec \alpha$	$-\text{cosec}\ \alpha$	$-\text{cotg}\ \alpha$
$90^o - \alpha$	$\text{cosec}\ \alpha$	$\sec \alpha$	$\text{tg}\ \alpha$
$360^o - \alpha$	$\sec \alpha$	$-\text{cosec}\ \alpha$	$-\text{cotg}\ \alpha$
$180^o + \alpha$	$-\sec \alpha$	$-\text{cosec}\ \alpha$	$\text{cotg}\ \alpha$

2.

Medida	rad	Ángulo 1.er cuadrante	seno	coseno	tangente	secante	cosecante	cotangente
120º	$\frac{2\pi}{3}$	120º = 180º – 60º	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$
135º	$\frac{3\pi}{4}$	135º = 180º – 45º	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
150º	$\frac{5\pi}{6}$	150º = 180º – 30º	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\frac{2\sqrt{3}}{3}$	2	$-\sqrt{3}$
210º	$\frac{7\pi}{6}$	210º = 180º + 30º	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$-\frac{2\sqrt{3}}{3}$	-2	$\sqrt{3}$
225º	$\frac{5\pi}{4}$	225º = 180º + 45º	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240º	$\frac{4\pi}{3}$	240º = 180º + 60º	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	-2	$-\frac{2\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$
315º	$\frac{7\pi}{4}$	315º = 360º – 45º	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
300º	$\frac{3\pi}{2}$	300º = 360º – 60º	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	2	$-\frac{2\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$
330º	$\frac{11\pi}{6}$	330º = 360º – 30º	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	-2	$-\sqrt{3}$

3. **a)** $\text{sen } 540^o = \text{sen } (360^o + 180^o) = \text{sen } 180^o = 0$

b) $\cos 390^o = \cos (360^o + 30^o) = \cos 30^o = \frac{\sqrt{3}}{2}$

c) $\text{tg } 750^o = \text{tg } (2 \cdot 360^o + 30^o) = \text{tg } 30^o = \frac{\sqrt{3}}{3}$

d) $\sec \frac{9\pi}{4} = \sec 405^o = \sec (360^o + 45^o) = \frac{\sqrt{2}}{2}$

e) $\text{cosec } 2190^o = \text{cosec } (360^o \cdot 6 + 30^o) = 2$

f) $\text{cotg } 600^o = \text{cotg } (360^o + 240^o) = \frac{1}{\sqrt{3}}$

g) $\text{sen } 1380^o = \text{sen } (360^o \cdot 4 - 60^o) = \frac{\sqrt{3}}{2}$

h) $\cos (-\frac{\pi}{4}) = \cos (-45^o) = \frac{\sqrt{2}}{2}$

i) $\text{cotg } (-150^o) = \text{cotg } (360 - 150) = \sqrt{3}$

j) $\sec 1260^o = \sec (360^o \cdot 3 + 180^o) = -1$

Ficha: Cálculo de razones a partir de otras conocidas

1. $\cos\alpha = \frac{\sqrt{15}}{4}$ $\quad \text{tg}\ \alpha = \frac{\sqrt{15}}{15}$

$\text{sen}\ \beta = -\frac{1}{2}$ $\quad \cos\beta = \frac{\sqrt{3}}{2}$ $\quad \text{tg}\ \beta = -\frac{\sqrt{3}}{3}$

$\cos\gamma = -\frac{4}{5}$ $\quad \text{tg}\ \gamma = \frac{3}{4}$

a) $\text{sen}\ 2\alpha = 2\cdot\frac{1}{4}\cdot\frac{\sqrt{15}}{4} = \frac{\sqrt{15}}{8}$

b) $\cos\frac{\beta}{2} = -\sqrt{\frac{1+\frac{\sqrt{15}}{4}}{2}} = -\sqrt{\frac{4+\sqrt{15}}{8}}$

c) $\text{tg}\ 2\gamma = \frac{2\left(-\frac{\sqrt{3}}{3}\right)}{1-\left(-\frac{\sqrt{3}}{3}\right)^2} = -\sqrt{3}$

d) $\text{sen}\ (\alpha+\beta) = \frac{1}{4}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{15}}{4}\left(-\frac{1}{2}\right) = \frac{\sqrt{3}-\sqrt{15}}{8}$

e) $\sec\frac{\gamma}{2} = \frac{1}{\cos\frac{\gamma}{2}} = -\sqrt{\frac{2}{1+\cos\gamma}} = -\sqrt{\frac{2}{1-\frac{4}{5}}} = -\sqrt{\frac{2}{\frac{1}{5}}} = -\sqrt{10}$

f) $\cos 3\beta = \cos(2\beta+\beta) = \cos 2\beta\cos\beta - \text{sen}\ 2\beta\,\text{sen}\ \beta = \frac{1}{2}\cdot\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) = 0$

$\cos 2\beta = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 = \frac{1}{2}$ $\qquad \text{sen}\ 2\beta = 2\cdot\left(-\frac{1}{2}\right)\cdot\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{2}$

g) $\cos(\gamma-\beta) = \left(-\frac{4}{5}\right)\cdot\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{3}{5}\right)\cdot\left(-\frac{1}{2}\right) = \frac{3-4\sqrt{3}}{10}$

h) $\text{cotg}\ \frac{\alpha}{2} = \frac{1}{\text{tg}\ \frac{\alpha}{2}} = \frac{1}{\sqrt{\frac{1-\frac{\sqrt{15}}{4}}{1+\frac{\sqrt{15}}{4}}}} = \sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}}}$

Ficha: Ecuaciones trigonométricas

1. **a)** $2\text{sen}\, x = 1 \Rightarrow \text{sen}\, x = \frac{1}{2} \Rightarrow x = \text{arcsen}\frac{1}{2} \Rightarrow x = 30^\circ,\ x = 150^\circ$

b) $1 - \cos x = \frac{1}{2} \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \arccos\frac{1}{2} \Rightarrow x = 60^\circ,\ \ x = 300^\circ$

c) $\cos x = 0 \Rightarrow x = \arccos 0 \Rightarrow x = 90^\circ,\ x = 270^\circ$

d) $\sqrt{3}\text{tg}\, x = -1 \Rightarrow \text{tg}\, x = \frac{-1}{\sqrt{3}} \Rightarrow x = \text{arctg}\left(\frac{-1}{\sqrt{3}}\right) \Rightarrow x = 150^\circ,\quad x = 330^\circ$

2. **a)** $\text{sen}^2 x = 1 \Rightarrow \begin{cases} \text{sen}\, x = 1 \\ \text{sen}\, x = -1 \end{cases} \Rightarrow \begin{cases} x = \frac{\pi}{2} + 2k\pi \\ x = \frac{3\pi}{2} + 2k\pi \end{cases}$ La solución puede expresarse como $x = \frac{\pi}{2} + k\pi,\ \ k \in \mathbb{Z}$

b) $\cos^2 x + 2\cos x = -1 \Rightarrow \cos^2 x + 2\cos x + 1 = 0 \qquad \cos x = t$

$t^2 + 2t + 1 = 0 \Rightarrow (t+1)^2 = 0 \Rightarrow t = -1 \Rightarrow \cos x = -1 \Rightarrow x = \pi + 2k\pi \Rightarrow x = (2k+1)\pi,\ k \in \mathbb{Z}$

c) $\text{tg}^2 x - 1 = 0 \Rightarrow \text{tg}^2 x = 1 \Rightarrow \begin{cases} \text{tg} x = 1 \\ \text{tg}\, x = -1 \end{cases} \Rightarrow \begin{cases} x = \frac{\pi}{4} + k\pi \\ x = \frac{3\pi}{4} + k\pi \end{cases} \quad k \in \mathbb{Z}$

d) $\cos^2 x + \frac{3}{2}\cos x - 1 = 0 \qquad \cos x = t$

$t^2 + \frac{3}{2}t - 1 = 0 \Rightarrow t = -2,\ t = \frac{1}{2} \Rightarrow \begin{cases} \cos x = -2 \ \text{ No existe ningún valor de x} \in \mathbb{R} \\ \cos x = \frac{1}{2} \Rightarrow x = \arccos\frac{1}{2} \Rightarrow x = \frac{\pi}{3} + 2k\pi \quad x = \frac{5\pi}{3} + 2k\pi \end{cases}$

3. **a)** $3\text{sen}^2x + 6\cos^2 x = 1 \Rightarrow 3\text{sen}^2x + 6(1-\text{sen}^2x) = 1 \Rightarrow -3\text{sen}^2x = -5$

$\Rightarrow \text{sen}^2x = \dfrac{5}{3} \Rightarrow \text{sen}x = \pm\sqrt{\dfrac{5}{3}}$ No tiene solución en $\mathbb{R}$

b) $\dfrac{3\text{sen}2x}{5\cos x} = \dfrac{1}{10} \Rightarrow \dfrac{6\text{sen}\, x\cos x}{5\cos x} = \dfrac{1}{10} \Rightarrow \dfrac{6}{5}\text{sen}\, x = \dfrac{1}{10} \Rightarrow \text{sen}\, x = \dfrac{1}{12} \Rightarrow \begin{cases} x \simeq 4^\circ 46'48'' + 360^\circ k \\ x \simeq 175^\circ 12'36'' + 360^\circ k \end{cases}$ $k \in \mathbb{Z}$

c) $\cos x\,\text{tg}\, x + \text{sen}\, x = -1 \Rightarrow \cos x \cdot \dfrac{\text{sen}\, x}{\cos x} + \text{sen}\, x = -1$

$\text{sen}\, x + \text{sen}\, x = -1 \Rightarrow \text{sen}\, x = \dfrac{-1}{2} \Rightarrow \begin{cases} x = 210^\circ + 360^\circ k \\ x = 330^\circ + 360^\circ k \end{cases}$ $k \in \mathbb{Z}$

d) $\sqrt{3}\cos x = \text{tg}\dfrac{x}{2} \Rightarrow \sqrt{3}\cos x = \sqrt{\dfrac{1-\cos x}{2}} \Rightarrow 3\cos^2 x = \dfrac{1-\cos x}{2} \Rightarrow 6\cos^2 x = 1 - \cos x \Rightarrow$

$\Rightarrow 6\cos^2 x + \cos x - 1 = 0 \qquad \cos x = t$

$6t^2 + t - 1 = 0 \Rightarrow t = \dfrac{-1}{2}, t = \dfrac{1}{3} \Rightarrow \begin{cases} \cos x = \dfrac{-1}{2} \\ \cos x = \dfrac{1}{3} \end{cases} \Rightarrow \begin{cases} x = 120^\circ + 360^\circ k \\ x = 240^\circ + 360^\circ k \\ x \simeq 70^\circ\ 31'\,12'' + 360^\circ k \\ x \simeq 109^\circ\ 28'\ 12'' + 360^\circ k \end{cases}$ $k \in \mathbb{Z}$

e) $\cos x\,\text{sen}\,2x - 2\cos 2x\text{sen}\, x = -\dfrac{1}{4} \Rightarrow 2\cos^2 x\text{sen}\, x - 2(\cos^2 x - \text{sen}^2 x)\text{sen}\, x = -\dfrac{1}{4} \Rightarrow$

$\Rightarrow 2\text{sen}^3x = -\dfrac{1}{4} \Rightarrow \text{sen}^3x = -\dfrac{1}{8} \Rightarrow \text{sen}x = \sqrt[3]{-\dfrac{1}{8}} = -\dfrac{1}{2} \Rightarrow \begin{cases} x = 210^\circ + 360^\circ k \\ x = 330^\circ + 360^\circ k \end{cases}$ $k \in \mathbb{Z}$

f) $\cos^2 x = 1 + \text{sen}^2 x \Rightarrow \cos^2 x - \text{sen}^2 x = 1 \Rightarrow \cos 2x = 1 \Rightarrow 2x = 360^\circ k \Rightarrow x = 180^\circ k,\ k \in \mathbb{Z}$

4. **a)** $\text{sen}x\cos x + \cos x = 0 \Rightarrow \cos x(\text{sen}x + 1) = 0 \Rightarrow \begin{cases} \cos x = 0 \\ \text{sen}x + 1 = 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x = \dfrac{\pi}{2} \\ \text{sen}x = -1 \text{ No hay solución en el primer cuadrante} \end{cases}$

b) $\text{sen}^2x + 4\text{sen}x = 0 \Rightarrow \text{sen}x(\text{sen}x + 4) = 0 \Rightarrow \begin{cases} \text{sen}x = 0 \Rightarrow x = 0^\circ \\ \text{sen}x = -4 \text{ No tiene solución} \end{cases}$

c) $15\text{sen}^3x - 8\text{sen}^2x = -\text{sen}x \Rightarrow 15\text{sen}^3x - 8\text{sen}^2x + \text{sen}x = 0 \Rightarrow \text{sen}x(15\text{sen}^2x - 8\text{sen}x + 1) = 0 \Rightarrow$

$\Rightarrow \begin{cases} \text{sen}x = 0 \Rightarrow x = 0^\circ \\ 15\text{sen}^2x - 8\text{sen}x + 1 = 0 \underset{t=\text{sen}x}{\Rightarrow} 15t^2 - 8t + 1 = 0 \Rightarrow t = \dfrac{1}{5}, t = \dfrac{1}{3} \Rightarrow \begin{cases} \text{sen}x = \dfrac{1}{5} \Rightarrow x \simeq 11^\circ\ 31'\ 48'' \\ \text{sen}x = \dfrac{1}{3} \Rightarrow x \simeq 19^\circ\ 28'\ 12'' \end{cases} \end{cases}$

5. **a)** $\begin{cases} x-y=0 \\ \cos^2 x-\operatorname{sen}^2 y=1 \end{cases} \Rightarrow \begin{cases} x=y \\ \cos^2 x-\operatorname{sen}^2 x=1 \end{cases} \Rightarrow \cos^2 x-\operatorname{sen}^2 x=1 \Rightarrow \cos 2x=1 \Rightarrow 2x=2k\pi \Rightarrow x=k\pi$

$\Rightarrow x=k\pi,\ y=k\pi \quad k\in\mathbb{Z}$

b) $2\cos x=\sqrt{3} \Rightarrow \cos x=\dfrac{\sqrt{3}}{2} \Rightarrow \begin{cases} x=\dfrac{\pi}{6}+2k\pi & k\in\mathbb{Z} \\ x=\dfrac{11\pi}{6}+2k\pi & k\in\mathbb{Z} \end{cases}$

$-2\cos y=\sqrt{3} \Rightarrow \cos y=\dfrac{-\sqrt{3}}{2} \Rightarrow \begin{cases} y=\dfrac{5\pi}{6}+2k\pi & k\in\mathbb{Z} \\ y=\dfrac{7\pi}{6}+2k\pi & k\in\mathbb{Z} \end{cases}$

c) $\begin{cases} y=2x \\ \operatorname{sen}x=2\operatorname{sen}y \end{cases} \Rightarrow \operatorname{sen}x=2\operatorname{sen}2x \Rightarrow \operatorname{sen}x-2\operatorname{sen}2x=0 \Rightarrow \operatorname{sen}x-4\operatorname{sen}x\cos x=0 \Rightarrow$

$\Rightarrow \operatorname{sen}x(1-4\cos x)=0 \Rightarrow \begin{cases} \operatorname{sen}x=0 \Rightarrow x=180^\circ k & k\in\mathbb{Z} \\ 1-4\cos x=0 \Rightarrow \cos x=\dfrac{1}{4} \Rightarrow \begin{cases} x=75{,}52^\circ+360^\circ k & k\in\mathbb{Z} \\ x=284{,}47^\circ+360^\circ k & k\in\mathbb{Z} \end{cases} \end{cases}$

d) $\begin{cases} \operatorname{tg}(x+y)=1 \\ \operatorname{tg}x+\operatorname{tg}y=1 \end{cases} \Rightarrow \begin{cases} \dfrac{\operatorname{tg}x+\operatorname{tg}y}{1-\operatorname{tg}x\operatorname{tg}y}=1 \\ \operatorname{tg}x+\operatorname{tg}y=1 \end{cases} \Rightarrow \dfrac{1}{1-\operatorname{tg}x\operatorname{tg}y}=1 \Rightarrow 1=1-\operatorname{tg}x\operatorname{tg}y \Rightarrow \operatorname{tg}x\operatorname{tg}y=0 \Rightarrow \begin{cases} \operatorname{tg}x=0 \\ \operatorname{tg}y=0 \end{cases} \Rightarrow$

$$\Rightarrow \begin{cases} \operatorname{tg}x=0 \Rightarrow x=k\pi \Rightarrow \operatorname{tg}y=1 \Rightarrow \begin{cases} y=\dfrac{\pi}{4}+2k\pi & k\in\mathbb{Z} \\ y=\dfrac{5\pi}{4}+2k\pi & k\in\mathbb{Z} \end{cases} \\ \operatorname{tg}y=0 \Rightarrow y=k\pi \Rightarrow \operatorname{tg}x=1 \Rightarrow \begin{cases} x=\dfrac{\pi}{4}+2k\pi & k\in\mathbb{Z} \\ x=\dfrac{5\pi}{4}+2k\pi & k\in\mathbb{Z} \end{cases} \end{cases}$$

Ficha: Resolución de triángulos

1. **a)** Llamamos *x* al cateto horizontal, e *y* a la hipotenusa.

Como tenemos la medida del cateto vertical, para hallar la medida del cateto horizontal utilizamos la tangente de 30º:

$$\text{tg}\,30º = \frac{10}{x} \Rightarrow x = \frac{10}{\text{tg}\,30º} = 17{,}32 \text{ cm}$$

Para hallar la medida de la hipotenusa, podemos utilizar el seno de 30º:

$$\text{sen}\,30º = \frac{10}{y} \Rightarrow y = \frac{10}{\text{sen}\,30º} = 20 \text{ cm}$$

b) Como el ángulo es de 45º, el triángulo es rectángulo e isósceles, es decir, los dos catetos son iguales. Llamamos *x* al cateto vertical, e *y* a la hipotenusa. Podemos aplicar las relaciones trigonométricas para comprobarlo y hallar la medida de la hipotenusa:

$$\text{tg}45º = \frac{x}{1{,}56} \Rightarrow x = 1{,}56 \cdot 1 = 1{,}56 \text{ m} \quad \cos 45º = \frac{1{,}56}{y} \Rightarrow y = \frac{1{,}56}{\cos 45º} = 2{,}21 \text{ m}$$

c) Como tenemos la medida de la hipotenusa, para hallar la medida de los catetos, llamamos *x* al horizontal, e *y* al vertical:

$$\cos 71º = \frac{x}{12{,}7} \Rightarrow x = 12{,}7 \cos 71º = 4{,}13 \text{ m} \qquad \text{sen}\,71º = \frac{y}{12{,}7} \Rightarrow y = 12{,}7\,\text{sen}\,71º = 12{,}008 \text{ m}$$

d) En este caso, el cateto horizontal, *x*, es el opuesto al ángulo de 38º, por lo que para hallarlo utilizamos el seno de 38º. Para el cateto vertical, *y*, utilizamos el coseno de 38º:

$$\text{sen}38º = \frac{x}{6{,}5} \Rightarrow x = 6{,}5\text{sen}38º = 4{,}001 \text{ m} \quad \cos\ 38º = \frac{y}{6{,}5} \Rightarrow y = 6{,}5 \cos 38º = 5{,}12 \text{ m}$$

2. **a)** $\text{tg}20º = \frac{10}{y} \Rightarrow y = \frac{10}{\text{tg}20º} \Rightarrow y = 27{,}47 \text{ cm}$

$$\text{tg}(x + 20º) = \frac{15 + 10}{y} = \frac{25}{27{,}47} = 0{,}91 \Rightarrow x + 20º \simeq 42{,}30º = 42º\ 18'\ 8'' \Rightarrow x = 42º\ 18'\ 8''$$

b) $\text{tg}\,6º = \frac{1}{y} \Rightarrow y = \frac{1}{\text{tg}\,6º} \Rightarrow y = 9{,}51 \text{ m}$

$$\text{tg}46º = \frac{x+1}{9{,}51} \Rightarrow x + 1 = 9{,}51\,\text{tg}46º = 9{,}85 \text{ cm} \Rightarrow x = 8{,}85 \text{ cm}$$

3. **a)** $a^2 = 90{,}1^2 + 100{,}2^2 - 2\cdot 90{,}1\cdot 100{,}2\cdot \cos 25^\circ = 1793{,}72 \Rightarrow a = 42{,}35$ m

$\frac{42{,}35}{\text{sen}\,25^\circ} = \frac{90{,}1}{\text{sen}\,\hat{B}} \Rightarrow \text{sen}\,\hat{B} = 0{,}899 \Rightarrow \hat{B} \simeq 64{,}04^\circ = 64^\circ\ 2'\ 42''$ $\quad \hat{C} = 180^\circ - (25^\circ + 64^\circ\ 2'\ 42'') = 90^\circ\ 57'\ 18''$

b) $\hat{C} = 180^\circ - (93^\circ + 75^\circ) = 12$

$\frac{13{,}1}{\text{sen}\,93^\circ} = \frac{a}{\text{sen}\,75^\circ} \Rightarrow a = \frac{13{,}1\cdot \text{sen}\,75^\circ}{\text{sen}\,93^\circ} = 12{,}67$ cm $\quad \frac{13{,}1}{\text{sen}\,93^\circ} = \frac{c}{\text{sen}\,12^\circ} \Rightarrow c = \frac{13{,}1\cdot \text{sen}\,12^\circ}{\text{sen}\,93^\circ} = 2{,}72$ cm

c) $\frac{10}{\text{sen}\,\hat{C}} = \frac{14{,}3}{\text{sen}\,54^\circ} \Rightarrow \text{sen}\,\hat{C} = \frac{10\,\text{sen}\,54^\circ}{14{,}3} = 0{,}56 \Rightarrow \hat{C} \simeq 34{,}06^\circ = 34^\circ\ 3'\ 36''$

$\hat{B} = 180^\circ - (54^\circ + 34^\circ\ 3'\ 36'') = 91^\circ\ 56'\ 24''$

$\frac{14{,}3}{\text{sen}\,54^\circ} = \frac{b}{\text{sen}(91^\circ\ 56'\ 24'')} \Rightarrow b = \frac{14{,}3\,\text{sen}(91^\circ\ 56'\ 24'')}{\text{sen}\,54^\circ} = 17{,}66$ m

d)

$40^2 = 60{,}9^2 + 92^2 - 2\cdot 60{,}9\cdot 92\cos\hat{A} \Rightarrow \cos\hat{A} = 0{,}94 \Rightarrow \hat{A} \simeq 19{,}34^\circ = 19^\circ 20'\ 24''$

$\frac{40}{\text{sen}\,19{,}34^\circ} = \frac{92}{\text{sen}\,\hat{C}} \Rightarrow \hat{C} \simeq 49{,}61^\circ = 49^\circ 36'\ 95''$ $\quad \hat{B} = 180^\circ - (19^\circ\ 20'\ 24'' + 49^\circ\ 36'\ 95'') = 111^\circ\ 2'\ 1''$

e)

$a^2 = 9^2 + 11^2 - 2\cdot 9\cdot 11\cos 108^\circ = 263{,}19 \Rightarrow a = 16{,}22$ m

$\frac{16{,}22}{\text{sen}\,108^\circ} = \frac{11}{\text{sen}\,\hat{B}} \Rightarrow \text{sen}\,\hat{B} = \frac{11\text{sen}\,108^\circ}{16{,}22} = 0{,}64 \Rightarrow \hat{B} \simeq 40{,}16^\circ = 40^\circ\ 9'\ 36''$

$\hat{C} = 180^\circ - (108^\circ + 40^\circ\ 9'\ 36'') = 31^\circ\ 50'\ 24''$

f) $2^2 = 1^2 + 1^2 - 2\cdot 1\cdot 1\cos\hat{A} \Rightarrow \cos\hat{A} = -1 \Rightarrow \hat{A} = 180^\circ$

No hay solución ya que las tres medidas no forman un triángulo.

PROFUNDIZACIÓN

Ficha: Ecuaciones trigonométricas más complicadas

1.

	1	0	0	b^3
$-b$		$-b$	b^2	$-b^3$
	1	$-b$	b^2	0

Se deduce que $(x^3+b^3)=(x+b)(x^2-bx+b)$.

Por tanto, si $x=a\Rightarrow a^3+b^3=(a+b)(a^2-ba+b^2)$

2. $\text{sen}^3 x+\cos^3 x=(\text{sen}x+\cos x)(\text{sen}^2 x-\text{sen}x\cos x+\cos^2 x)=(\text{sen}x+\cos x)(1-\text{sen}x\cos x)$

3. $\text{sen}^3 x+\cos^3 x+\text{sen}x\cos x=1\Rightarrow \text{sen}^3 x+\cos^3 x+\text{sen}x\cos x-1=0\Rightarrow$
$\Rightarrow(\text{sen}x+\cos x)(1-\text{sen}x\cos x)+\text{sen}x\cos x-1\Rightarrow$
$\Rightarrow(\text{sen}x+\cos x-1)(1-\text{sen}x\cos x)=0$

Por tanto, la ecuación se reduce a resolver las ecuaciones: $1-\text{sen}x\cos x=0$ y $\text{sen}x+\cos x-1=0$

$1-\text{sen}\,x\cos x=0\Rightarrow \text{sen}\,x\cos x=1\Rightarrow 2\,\text{sen}\,x\cos x=2\Rightarrow \text{sen}\,2x=2\Rightarrow$ No tiene solución.

$\text{sen}\,x+\cos x=1\Rightarrow(\text{sen}\,x+\cos x)^2=\text{sen}^2 x+\cos^2 x+2\,\text{sen}\,x\cos x\Rightarrow$

$\Rightarrow 1^2=1+\text{sen}\,2x\Rightarrow \text{sen}\,2x=0\Rightarrow 2x=k\pi\Rightarrow x=k\cdot\dfrac{\pi}{2}$

4. $(a^2+b^2)^2=a^4+b^4+2a^2b^2\Rightarrow a^4+b^4=(a^2+b^2)^2-2a^2b^2$

5. $\text{sen}^4 x+\cos^4 x=(\text{sen}^2 x+\cos^2 x)^2-2\,\text{sen}^2 x\cos^2 x=1^2-\dfrac{1}{2}\cdot 4\,\text{sen}^2 x\cos^2 x=1-\dfrac{1}{2}(2\,\text{sen}\,x\cos x)^2=1-\dfrac{\text{sen}^2 2x}{2}$

6. a) $\text{sen}^4 x+\cos^4 x=1\Rightarrow 1-\dfrac{\text{sen}^2 2x}{2}=1\Rightarrow -\dfrac{\text{sen}^2 2x}{2}=0\Rightarrow \text{sen}^2 2x=0\Rightarrow \text{sen}\,2x=0\Rightarrow 2x=k\pi\Rightarrow x=\dfrac{k\pi}{2}$

b) $\text{sen}^4 x+\cos^4 x=0\Rightarrow 1-\dfrac{\text{sen}^2 2x}{2}=0\Rightarrow \dfrac{\text{sen}^2 2x}{2}=1\Rightarrow \text{sen}^2 2x=2$

Como $0\le \text{sen}^2\alpha\le 1$ para cualquier ángulo α, la ecuación no tiene solución.

c) $\text{sen}^4\left(\dfrac{x}{5}\right)+\cos^4\left(\dfrac{x}{5}\right)=\dfrac{5}{8}\Rightarrow 1-\dfrac{\text{sen}^2\left(\frac{2x}{5}\right)}{2}=\dfrac{5}{8}\Rightarrow \dfrac{\text{sen}^2\left(\frac{2x}{5}\right)}{2}=\dfrac{3}{8}\Rightarrow \text{sen}^2\left(\dfrac{2x}{5}\right)=\dfrac{3}{4}\Rightarrow \text{sen}\left(\dfrac{2x}{5}\right)=\pm\dfrac{\sqrt{3}}{2}\Rightarrow$

$$\Rightarrow\begin{cases}\dfrac{2x}{5}=\dfrac{\pi}{3}+2k\pi & \dfrac{2x}{5}=\dfrac{4\pi}{3}+2k\pi\\[2mm] \dfrac{2x}{5}=\dfrac{2\pi}{3}+2k\pi & \dfrac{2x}{5}=\dfrac{5\pi}{3}+2k\pi\end{cases}\Rightarrow\begin{cases}x=\dfrac{5\pi}{6}+5k\pi & x=\dfrac{10\pi}{3}+5k\pi\\[2mm] x=\dfrac{5\pi}{3}+5k\pi & x=\dfrac{25\pi}{6}+5k\end{cases}$$

7. $\text{sen}^4 x+\cos^4 x=k\Rightarrow 1-\dfrac{\text{sen}^2 2x}{2}=k\Rightarrow \text{sen}^2 2x=2-2k\Rightarrow 0\le 2-2k\le 1\Rightarrow -2\le -2k\le -1\Rightarrow$

$\Rightarrow 2\ge 2k\ge 1\Rightarrow 1\ge k\ge\dfrac{1}{2}\Rightarrow k\in\left[\dfrac{1}{2},\,1\right]$

Ficha: Perímetro y área

a) $\cos\alpha = \frac{PB}{PQ} \Rightarrow PQ = \frac{PB}{\cos\alpha} = \frac{5}{\cos\alpha}$

$\text{tg}\,\alpha = \frac{BQ}{PB} \Rightarrow BQ = PB\ \text{tg}\,\alpha = 5\,\text{tg}\,\alpha$

$BQ + QR + RC = 5 \Rightarrow QR = 5 - 2BQ = 5 - 2\cdot 5\,\text{tg}\,\alpha = 5 - 10\,\text{tg}\,\alpha$

Perímetro: $4PQ + 2QR = 4\cdot\frac{5}{\cos\alpha} + 2(5 - 10\,\text{tg}\,\alpha) = \frac{20}{\cos\alpha} + 10 - 20\,\text{tg}\,\alpha$

b) $\alpha = \frac{\pi}{5} \Rightarrow P = \frac{20}{\cos\left(\frac{\pi}{5}\right)} + 10 - 20\,\text{tg}\left(\frac{\pi}{5}\right) = 20{,}190 \text{ cm}$

$\alpha = \frac{\pi}{6} \Rightarrow P = \frac{20}{\cos\left(\frac{\pi}{6}\right)} + 10 - 20\,\text{tg}\left(\frac{\pi}{6}\right) = 21{,}547 \text{ cm}$

$\alpha = \frac{\pi}{8} \Rightarrow P = \frac{20}{\cos\left(\frac{\pi}{8}\right)} + 10 - 20\,\text{tg}\left(\frac{\pi}{8}\right) = 23{,}364 \text{ cm}$

c) $\frac{20}{\cos\alpha} + 10 - 20\,\text{tg}\,\alpha = 30\cdot\frac{3}{4} \Rightarrow \frac{20}{\cos\alpha} + 10 - 20\,\text{tg}\,\alpha = \frac{45}{2} \Rightarrow 40 + 20\cos\alpha - 40\,\text{tg}\,\alpha\cos\alpha = 45\cos\alpha \Rightarrow$

$\Rightarrow 25\cos\alpha + 40\,\text{sen}\,\alpha - 40 = 0 \Rightarrow 5\cos\alpha + 8\,\text{sen}\,\alpha - 8 = 0 \Rightarrow$

$\Rightarrow 5\sqrt{1-\text{sen}^2\alpha} + 8\,\text{sen}\,\alpha - 8 = 0 \Rightarrow 25(1 - \text{sen}^2\alpha) = 64 + 64\,\text{sen}^2\alpha - 128\,\text{sen}\,\alpha \Rightarrow$

$\Rightarrow 89\,\text{sen}^2\alpha - 128\,\text{sen}\,\alpha + 39 = 0 \Rightarrow \text{sen}\,\alpha = \frac{128 \pm 50}{178} \Rightarrow \begin{cases} \text{sen}\,\alpha = 1 \\ \text{sen}\,\alpha = \frac{39}{89} \end{cases}$

$\Rightarrow \alpha = \text{arcsen}\frac{39}{89} = 0{,}4536 \text{ rad} = 25^\circ\ 59'$

d) El área de cada uno de los triángulos que aparecen en la figura es:

$$\frac{5BQ}{2} = \frac{5\cdot 5\,\text{tg}\,\alpha}{2} = \frac{25}{2}\text{tg}\,\alpha$$

El área del hexágono será $S = 5\cdot 10 - 4\cdot\frac{25}{2}\text{tg}\,\alpha = 50 - 50\,\text{tg}\,\alpha = 50(1 - \text{tg}\,\alpha)$.

e) $50(1 - \text{tg}\,\alpha) = \frac{3}{4}\cdot 50 \Rightarrow 1 - \text{tg}\,\alpha = \frac{3}{4} \Rightarrow \text{tg}\,\alpha = \frac{1}{4} \Rightarrow \alpha = \text{arctg}\left(\frac{1}{4}\right) = 0{,}245 \text{ rad} = 14^\circ\ 2'$